

Semileptonic and rare B meson decays into a light pseudoscalar meson

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Abstract

In the framework of a QCD relativistic potential model we evaluate the form factors describing the exclusive decays $B \rightarrow \pi \ell \nu$ and $B \rightarrow K \ell^+ \ell^-$. The present calculation extends a previous analysis of B meson decays into light vector mesons. We find results in agreement with the data, when available, and with the theoretical constraints imposed by the Callan-Treiman relation and the infinite heavy quark mass limit.

The study of the decays

$$B \rightarrow \pi \ell \bar{\nu}_\ell \tag{1}$$

$$B \rightarrow K \ell^+ \ell^- \tag{2}$$

represents a significant part of the experimental programmes at the next proton-proton accelerators and at the future B -factories at SLAC and KEK. The importance of these processes arises from the following reasons. The decay (1) allows to measure the product of the Kobayashi-Maskawa (KM) matrix element V_{ub} and the form factor describing the decay process* ; similarly, the decay (2) will give access, in appropriate regions of phase space, to the KM matrix element V_{ts} ; therefore these processes would allow to measure fundamental parameters of the *Standard Model* (SM) of the fundamental interactions, to say nothing of the possibility to explore, in both cases, new effects beyond the SM .

It is fair to say, however, that, in spite of the fundamental relevance of the processes (1) and (2), the basic theory of the hadronic interactions, Quantum-Chromo-Dynamics (QCD), is still unable to produce clear predictions for the hadronic matrix elements $B \rightarrow \pi$, $B \rightarrow K$ involved in these decays. This is due to the lack of a theoretical tool, as powerful as perturbation theory, able to produce predictions for the nonperturbative quantities involved in these processes. The most frequently used theoretical methods to deal with these problems are based on approximation schemes such as lattice QCD or QCD sum rules. These approaches have however their own limitations. In the former method the finite lattice size introduces a cut-off in the small momenta, which precludes the possibility to make reliable predictions in the small momentum transfer region ($Q^2 \leq 15 \text{ GeV}^2$) (for recent reviews of lattice QCD predictions for B into light meson transitions see e.g. [1]). In the case of QCD sum rules or their variant, light cone sum rules, the theoretical uncertainties are dominated by the peculiar theoretical tools employed by this method (criteria for stability, hierarchic role of the different nonperturbative contributions parametrized by the various condensates) and cannot be reduced by adding new terms in the Operator Product Expansion (for a discussion see [2]).

On the basis of these considerations, in [3] we have presented an analysis of semileptonic and rare transitions between the meson B and a light vector meson in a QCD relativistic potential model. In [3] we argued that, because of its simplicity, this model might be used as a viable alternative to the more fundamental, but still limited theoretical approaches we have discussed above. It is the aim of this paper to extend this analysis to the decays (1) and (2).

To begin with, we review the main features of the QCD relativistic potential model. It is a *potential model* because the mesons are described as bound states of constituent quarks and antiquarks tied by an instantaneous potential $V(r)$. It is a *QCD model* because the potential is modelled according to the theory of the hadronic interactions, i.e. it has a confining linear behaviour at large interquark distances r and a Coulombic behaviour $\simeq -\alpha_s(r)/r$ at small distances, with $\alpha_s(r)$ the running strong coupling constant: in practice the interpolating Richardson's potential $V(r)$ is used [4], cut-off at very small distances (of the order of the inverse heavy meson mass)

*There is one form factor contributing to (1) for a massless lepton.

to take care of unphysical singularities introduced by the relativistic kinematics [5]. Finally it is a *relativistic model* because the wave equation used to obtain the meson wave function Ψ is the Salpeter equation embodying the relativistic kinematics:

$$\left[\sqrt{-\nabla^2 + m_1^2} + \sqrt{-\nabla^2 + m_2^2} + V(r) \right] \Psi(\vec{r}) = M\Psi(\vec{r}) \quad , \quad (3)$$

where, for heavy mesons made up by a heavy quark Q and a light antiquark, 1 refers the heavy quark and 2 to the light antiquark. The relativistic kinematics plays an important role when at least one of the two quarks constituting the meson is light, as in our case, and represents an improvement in comparison with the approach based on the non-relativistic quark model. In (3) M is the heavy meson mass that is obtained by fitting the various parameters of the model, in particular the b-quark mass, that is fitted to the value $m_b = 4890$ MeV, and the light quark masses $m_u \simeq m_d = 38$ MeV, $m_s = 115$ MeV[†]. The B -meson wave function in its rest frame is obtained by solving (3); a useful representation in the momentum space was obtained in [3] and is as follows

$$\psi(k) = 4\pi\sqrt{m_B\alpha^3} e^{-\alpha k} \quad , \quad (4)$$

with $\alpha = 2.4$ GeV⁻¹ and $k = |\vec{k}|$ the quark momentum in the B rest frame.

The constituent quark picture used in the model is well suited for the mesons comprising at least a heavy quark; for light mesons other dynamical features, not accounted for by this simple picture, should be incorporated, e.g. the nature of pseudo Nambu-Goldstone bosons of π 's and K 's and the presence of important *spin-spin* terms in $V(r)$, not included in the Richardson's potential (their neglect for heavy mesons is justified by the spin symmetry of the Heavy Quark Effective Theory (HQET)[6] which is valid in the limit $m_Q \rightarrow \infty$). The solution adopted in [3] was to avoid, for light mesons, the constituent quark picture and to describe their couplings to the quark degrees of freedom by effective vertices. This assumption produces a set of rules that are used to compute the quark loop of fig. 1, i.e. the diagram by which the hadronic amplitudes describing the decays (1) and (2) are evaluated. They are as follows.

1) For a light pseudoscalar meson M ($= \pi^\pm, K$) of momentum p' we write the coupling

$$- \frac{N_q N_{q'}}{f_M} \not{p}' \gamma_5 \quad , \quad (5)$$

where $f_M = f_\pi = 130$ MeV or $f_M = f_K = 160$ MeV. The normalization factors $N_q, N_{q'}$ for the quark coupled to the meson are discussed below.

2) For the heavy meson B in the initial state one introduces the matrix:

$$B = \frac{1}{\sqrt{3}} \psi(k) \sqrt{\frac{m_q m_b}{m_q m_b + q_1 \cdot q_2}} \frac{\not{q}_1 + m_b}{2m_b} (-i \gamma_5) \frac{-\not{q}_2 + m_q}{2m_q} \quad (6)$$

[†]Data on the heavy meson spectra are not of great help in fitting light quark masses, which, therefore, are not accurately determined in the model; its predictions, however, are not sensitive to m_u, m_d, m_s values in most of the available kinematical range.

where m_b and m_q are the heavy and light quark masses, q_1^μ , q_2^μ their 4-momenta. The normalization factor corresponds to the normalization $\langle B|B \rangle = 2 m_B$ and $\int \frac{d^3k}{(2\pi)^3} |\psi(k)|^2 = 2m_B$ already embodied in (6). One assumes that the 4-momentum is conserved at the vertex $B\bar{q}b$, i.e. $q_1^\mu + q_2^\mu = p^\mu = B$ meson 4-momentum. Therefore $q_1^\mu = (E_b, \vec{k})$, $q_2^\mu = (E_q, -\vec{k})$ and

$$E_b + E_q = m_B \quad . \quad (7)$$

3) To take into account the off-shell effects due to the quarks interacting in the meson, one introduces running quark masses $m(k)$, to enforce the condition

$$E = \sqrt{m^2(k) + |\vec{k}|^2} \quad (8)$$

for the constituent quarks. For the kinematics of the decays (1) and (2) it is sufficient to introduce the running mass only for the heavy quark [‡]

$$m_b = m_b(k) \quad , \quad (9)$$

defined by the condition

$$\sqrt{m_q^2 + |\vec{k}|^2} + \sqrt{m_b^2 + |\vec{k}|^2} = m_B \quad . \quad (10)$$

4) The condition $m_b^2 \geq 0$ implies the constraint

$$0 \leq k \leq k_M = \frac{m_B^2 - m_q^2}{2m_B} \quad , \quad (11)$$

on the integration over the loop momentum k

$$\int \frac{d^3k}{(2\pi)^3} \quad . \quad (12)$$

5) For each quark line with momentum q and not representing a constituent quark one introduces the factor

$$\frac{i}{\not{q} - m_{q'}} \times G(q^2) \quad , \quad (13)$$

where $G(q^2)$ is a shape function that modifies the free propagation of the quark of mass $m_{q'}$ in the hadronic matter. The shape function

$$G(q^2) = \frac{m_G^2 - m_{q'}^2}{m_G^2 - q^2} \quad (14)$$

was adopted in [3]; the value of the mass parameter m_G was determined in [3] by the experimental data on the $B \rightarrow K^*\gamma$ decay. A range $[1.2, 7.6]$ GeV² of possible values of m_G^2 was obtained.

6) For the hadronic current in fig. 1 one puts the factor

$$N_q N_{q'} \Gamma^\mu \quad , \quad (15)$$

[‡]By this choice, the average $\langle m_b(k) \rangle$ does not differ significantly from the value m_b fitted from the spectrum, see [3] for details.

where Γ^μ is a 4×4 matrix. We shall consider $\Gamma^\mu = \gamma^\mu$ and $\Gamma^\mu = \sigma^{\mu\nu} q_\nu$ (with $\sigma^{\mu\nu} = i/2 [\gamma^\mu, \gamma^\nu]$). The normalization factor N_q is as follows:

$$N_q = \begin{cases} \sqrt{\frac{m_q}{E_q}} & (\text{if } q = \text{constituent quark}) \\ 1 & (\text{otherwise}). \end{cases} \quad (16)$$

7) For each quark loop one puts a colour factor of 3 and performs a trace over Dirac matrices.

This set of rules can now be applied to the evaluation of the matrix element $\langle M(p') | \bar{q} \Gamma^\mu b | B(p) \rangle$ with the result:

$$\begin{aligned} \langle M(p') | \bar{q} \Gamma^\mu b | B(p) \rangle &= \sqrt{3} \int \frac{d^3 k}{(2\pi)^3} \theta[k_M - k] \psi(k) \sqrt{\frac{m_q m_b}{m_q m_b + q_1 \cdot q_2}} \\ & \text{Tr} \left[\frac{\not{q}_1 + m_b}{2m_b} (-i \gamma_5) \frac{\not{q}_2 + m_q}{2m_q} \sqrt{\frac{m_b m_q}{E_b E_q}} \frac{1}{f_M} (\not{p} - \not{q}) \gamma_5 \frac{i G[(q_1 - q)^2]}{\not{q}_1 - \not{q} - m_{q'} + i \epsilon} \Gamma^\mu \right]. \end{aligned} \quad (17)$$

From this expression one can obtain the relevant formulae for the various form factors. With $q = p - p'$, we write

$$\begin{aligned} \langle M(p') | \bar{q} \gamma^\mu b | \bar{B}(p) \rangle &= f_+(q^2)(p + p')^\mu + f_-(q^2)q^\mu \\ &= F_1(q^2)(p + p')^\mu + \frac{m_B^2 - m_M^2}{q^2} q^\mu (F_0(q^2) - F_1(q^2)) \quad , \end{aligned} \quad (18)$$

$$\langle M(p') | i \bar{q} \sigma^{\mu\nu} q_\nu b | \bar{B}(p) \rangle = \frac{f_T(q^2)}{m_B + m_M} \left[(p + p')^\mu q^2 - (m_B^2 - m_M^2) q^\mu \right] \quad , \quad (19)$$

where

$$\begin{aligned} F_1(q^2) &= f_+(q^2) \\ F_0(q^2) &= f_+(q^2) + \frac{q^2}{m_B^2 - m_M^2} f_-(q^2) \quad . \end{aligned} \quad (20)$$

In (18) and (19) we shall consider $M = \pi$ or $M = K$ since both cases are of physical interest if we wish to consider not only semileptonic and radiative transitions, but also nonleptonic decays.

The calculation of the trace and the integral in (17) is straightforward and is similar to the one obtained in [3] for $B \rightarrow \rho$, $B \rightarrow K^*$ transitions. For all the form factors we write $F(q^2) = F(q^2, m_{q'}) - F(q^2, m_G)$, where, for the various form factors, we have

$$\begin{aligned} F_0(q^2, x) &= \frac{\sqrt{6}}{4\pi^2 f_M (m_B^2 - m_M^2)} \int_0^{k_M} \frac{dk \quad k^2 \psi(k)}{\sqrt{E_q E_b [m_B^2 - (m_b - m_q)^2]}} \times \\ & \int_{-1}^1 dz \frac{1}{m_M^2 - 2E_q(m_B - q^0) + m_q^2 - x^2 + 2|\vec{q}| k z} \end{aligned}$$

$$\left\{ [2E_q(m_B - q^0) - m_M^2 - 2|\vec{q}| k z] [(m_q E_b + m_b E_q) q^0 + (m_b - m_q) |\vec{q}| k z] + \frac{m_q + m_{q'}}{2} \left[(q^2 - m_B q^0) [m_B^2 - (m_b - m_q)^2] + 2 m_B^2 |\vec{q}| k z \right] \right\} , \quad (21)$$

$$\begin{aligned} F_1(q^2, x) = & \frac{\sqrt{6}}{8\pi^2 f_M} \int_0^{k_M} \frac{dk \ k^2 \psi(k)}{\sqrt{E_q E_b [m_B^2 - (m_b - m_q)^2]}} \times \\ & \int_{-1}^1 dz \frac{1}{m_M^2 - 2E_q(m_B - q^0) + m_q^2 - x^2 + 2|\vec{q}| k z} \\ & \left\{ [2E_q(m_B - q^0) - m_M^2 - 2|\vec{q}| k z] \frac{|\vec{q}| (m_q E_b + m_b E_q) + q^0 k z (m_b - m_q)}{m_B |\vec{q}|} + \right. \\ & \left. \frac{m_q + m_{q'}}{2} \frac{2 k z (m_B q^0 - q^2) - |\vec{q}| [m_B^2 - (m_b - m_q)^2]}{|\vec{q}|} \right\} , \quad (22) \end{aligned}$$

$$\begin{aligned} f_T(q^2, x) = & -\frac{\sqrt{6}}{4\pi^2 f_M} \frac{m_B + m_M}{2} \int_0^{k_M} \frac{dk \ k^2 \psi(k)}{\sqrt{E_q E_b [m_B^2 - (m_b - m_q)^2]}} \times \\ & \int_{-1}^1 dz \frac{1}{m_M^2 - 2E_q(m_B - q^0) + m_q^2 - x^2 + 2|\vec{q}| k z} \\ & \left\{ [2E_q(m_B - q^0) - m_M^2 - 2|\vec{q}| k z] \frac{k z}{|\vec{q}|} + \right. \\ & \left. (m_q + m_{q'}) \frac{-(m_b E_q + m_q E_b) |\vec{q}| + (m_B - q^0) k z (m_b - m_q)}{m_B |\vec{q}|} \right\} . \quad (23) \end{aligned}$$

In these equations q_0 is the time component of four-momentum q^μ ,

$$z = \cos(\theta) , \quad (24)$$

with θ the angle between \vec{k} and the direction of transferred momentum \vec{q} . We note that, for $M = \pi$, $m_q = m_{q'} = m_u$ and $f_M = f_\pi$, while, for $M = K$, $m_q = m_u$, $m_{q'} = m_s$ and $f_M = f_K$.

Before discussing our numerical results in detail let us compute $F_0(q^2)$ for $q^2 = m_B^2 - m_M^2$; in the chiral limit $F_0(m_B^2 - m_M^2) \simeq F_0(m_B^2)$ must obey the *Callan - Treiman* relation [7]

$$F_0(m_B^2) = \frac{f_B}{f_\pi} . \quad (25)$$

This is therefore a consistency test to be satisfied by the model. We have numerically evaluated $F_0^{B\pi}(m_B^2)$ for different values of the parameter m_G and we have obtained the result $F_0^{B\pi}(m_B^2) \simeq 1.48$, almost independent of m_G . This result should be compared to $f_B/f_\pi \simeq 1.58$, which is obtained using $f_B = 0.2 \text{ GeV}$, i.e. the value computed in [3] using the present model. The small discrepancy

in the Callan-Treiman relation ($\simeq 6\%$) may be attributed to the deviations induced in the B meson wave function by the chiral limit that are not accounted for by this calculation. We expect however that these differences vanish if, in addition to the chiral limit, one also takes the infinite heavy quark mass limit; as a matter of fact one can verify rather easily, using the previous formula for F_0 and the expression in [3] for f_B , that the Callan-Treiman exactly holds in the combined $m_b \rightarrow \infty$ and $m_M \rightarrow 0$ limit.

Let us now consider the form factor $F_1(q^2)$ (respectively $f_T(q^2)$). Our numerical results for the central value of m_G , i.e. $m_G = 1.77$ GeV, show that the q^2 -behaviour of this form factor is increasing (resp. decreasing) for both small and moderate values of q^2 , independently of the value of the mass parameter m_G introduced in eq.(13). This behaviour should hold also at large q^2 ($q^2 \geq 15$ GeV²) due to the effect, in this region, of a pole in the q^2 functional dependence, predicted by the dispersion relation. Differently from our analysis of $F_0(q^2)$, we cannot pretend, however, to extend the validity of our predictions for $F_1(q^2)$ and $f_T(q^2)$ at the extreme values of q^2 . The difference between the two cases is as follows. In the case of F_0 , the pole (with $J^P = 0^+$) contribution to this form factor vanishes in the chiral limit and has therefore a minor impact on the q^2 behaviour. On the contrary the form factors F_1 and f_T , have a non vanishing polar contribution which becomes larger and larger with increasing q^2 . While we expect that this behaviour become visible well before the pole, at extreme values of q^2 the diverging behaviour induced by such a contribution cannot be reproduced by the model. As a matter of fact, for larger values of q^2 , $|\vec{q}|$ becomes smaller and smaller, and, therefore, the model becomes sensitive to the actual values of the parameters, in particular the light quark masses that are not accurately fitted by the available experimental data (see above). Therefore we can consider that our predictions are reliable in the range (0, 15) GeV²; at $q^2 = 0$ we get

$$\begin{aligned} F_0^{B\pi}(0) &= F_1^{B\pi}(0) = 0.37 \pm 0.12 \\ F_0^{BK}(0) &= F_1^{BK}(0) = 0.26 \pm 0.08 \\ f_T^{B\pi}(0) &= -0.14 \pm 0.02 \\ f_T^{BK}(0) &= -0.09^{+0.05}_{-0.02} . \end{aligned} \tag{26}$$

The central values are obtained for $m_G = 1.77$ GeV, which is the best fit of the parameter m_G found in [3] by the experimental branching ratio $\mathcal{B}(B \rightarrow K^*\gamma)$, whereas the theoretical uncertainty is obtained by varying m_G in the range [1.1, 2.8] GeV. The results for $B \rightarrow \pi$ refer to charged pions.

Let us now consider the q^2 -behaviour of the form factors. We introduce the two-parameter function for the three form factors

$$F(q^2) = \frac{F(0)}{1 - a_F \left(\frac{q^2}{m_B^2} \right) + b_F \left(\frac{q^2}{m_B^2} \right)^2} ; \tag{27}$$

here a_F, b_F are parameters to be fitted by means of the numerical analysis and $F(0)$ is given in eq. (26); to allow a comparison with other approaches we perform the analysis up to $q^2 = 15$ GeV²,

both for π and K mesons. We collect the fitted values in table 1 and report the q^2 -dependence in fig. 2, 3.

	$F(0)$	a_F	b_F	$F(0)$	a_F	b_F	
$F_1^{B\pi}$	0.37	0.60	0.065	0.26	0.50	0.39	F_1^{BK}
$F_0^{B\pi}$	0.37	1.1	0.44	0.26	1.2	0.56	F_0^{BK}
$f_T^{B\pi}$	-0.14	0.92	0.21	-0.09	0.76	0.76	f_T^{BK}

Table 1: Parameters appearing in eq. (27) for different B form factors.

From table 1 and from fig. 2, 3, one can see that $F_0(q^2)$, $F_1^{B\pi}(q^2)$ and $f_T^{B\pi}(q^2)$ have a q^2 behaviour similar to a single pole. For $F_1^{BK}(q^2)$ and $f_T^{BK}(q^2)$ there are significant deviations from this behaviour. We do not have yet experimental data to test these predictions and we shall limit to compare our results with other theoretical approaches; before doing that, let us discuss the infinite heavy quark mass limit of the model. In the framework of the Heavy Quark Effective Theory, which corresponds to the $m_b \rightarrow \infty$ limit, there is a constraint to be satisfied by the three form factors, i.e., the relation originally found in [8]:

$$f_T(q^2) = -\frac{m_B + m_M}{2 m_B} \left[F_1(q^2) - (m_B^2 - m_M^2) \frac{F_0(q^2) - F_1(q^2)}{q^2} \right]. \quad (28)$$

This relation holds in the limit $m_b \rightarrow \infty$ and for high q^2 ($q^2 \simeq q_{max}^2$). We have checked that this relation formally holds in our model in the infinite heavy quark mass limit and for $q^2 \simeq q_{max}^2$. For the actual value of m_B ($= 5.28$ GeV) and for the transition $B \rightarrow \pi$ the situation is as follows. We choose $q_M^2 = 15$ GeV², i.e. the maximum value at which we can trust our predictions and we find numerically $F_1(q_M^2) \simeq 0.54$, $F_0(q_M^2) \simeq 0.71$, $f_T(q_M^2) \simeq -0.27$. Therefore the relation (28) has a significant violation of 50%, that may be attributed to the fact that we are still far from q_{max}^2 and $\mathcal{O}(1/m_b)$ corrections are large. Similar results are obtained for the $B \rightarrow K$ transition.

Let us finally comment on the scaling laws of the form factors at large q^2 that can be helpful in using the heavy flavour symmetry to relate the form factors of B and D mesons [10]. From eqs.(22) and (23) the following behaviours can be formally derived:

$$F_1(q_{max}^2) \approx \sqrt{m_b} \quad (29)$$

$$f_T(q_{max}^2) \approx \sqrt{m_b}; \quad (30)$$

they are in agreement with the pole (vector meson) dominance of the form factors observed in [9] [10]; moreover in the chiral limit, from eq.(21) one gets:

$$F_0(q_{max}^2) \approx \frac{1}{\sqrt{m_b}}. \quad (31)$$

Let us now compare our work with other theoretical approaches. In table 2 we compare our outcome for the values at $q^2 = 0$ with the results of QCD sum rules and lattice QCD calculations

(for other work on this subject see, e.g. [11]). We observe that our results are in agreement, within the theoretical uncertainties, with the determinations obtained by light cone sum rules (*LCSR*) [12], lattice [1] and lattice + *LCSR* [14].

	<i>This work</i>	<i>LCSR</i> [12]	<i>SR</i> [13]	<i>Latt.</i> [1]	<i>Latt.</i> + LCSR [14]
$F_1^{B\pi}(0)$	0.37 ± 0.12	0.30 ± 0.04	0.24	0.27 ± 0.11	0.27 ± 0.11
$f_T^{B\pi}(0)$	-0.14 ± 0.02	-0.19 ± 0.02	---	---	---
$F_1^{BK}(0)$	0.26 ± 0.08	0.35 ± 0.05	0.25	---	---
$f_T^{BK}(0)$	$-0.09^{+0.05}_{-0.02}$	-0.15 ± 0.02	-0.14	---	---

Table 2: Comparison of the results coming from different approach to evaluate form factors.

As for the q^2 dependence, we have not reported the predictions of other theoretical approaches, since they qualitatively agree with our calculations. In absence of detailed experimental data on the form factors, the best we can do to test the model is to use data on the partial width $\Gamma(B \rightarrow \pi \ell \nu)$. To perform this comparison we must, however, extrapolate the q^2 -behaviour obtained by eq. (27) and tables 1 and 2, and valid in the region $(0, 15) \text{ GeV}^2$, to the whole q^2 range. This procedure implies an uncertainty which is difficult to assess, but should not be extremely large due to the phase space limitation at high q^2 . We obtain

$$\mathcal{BR}(\bar{B}^0 \rightarrow \pi^+ \ell^- \bar{\nu}) = 1.03 \left(\frac{|V_{ub}|}{3.2 \cdot 10^{-3}} \right)^2 10^{-4}, \quad (32)$$

to be compared to the experimental value $\mathcal{BR}(B \rightarrow \pi \ell \nu)_{exp.} = (1.8 \pm 0.6) \cdot 10^{-4}$ [15]. Therefore our result is compatible with the present range of the *KM* matrix element $V_{ub} = [1.8, 4.5] \times 10^{-3}$; the preferred range of values selected by the model and by the present experimental limits on V_{ub} is $V_{ub} = (4.0 \pm 0.5) \times 10^{-3}$.

We conclude our analysis by summarizing our results. We have used a QCD relativistic potential model, introduced in [3], to study the weak and radiative transitions $B \rightarrow \pi, K$. We have computed the relevant form factors and tested the Callan-Treiman and the Isgur-Wise relation. The former relation, valid in the chiral limit, is satisfied at the 6% level, while the latter, valid in the $m_b \rightarrow \infty$ limit, has significant violations, due to $\mathcal{O}(1/m_b)$ corrections. Our result for the branching ratio $\mathcal{BR}(B \rightarrow \pi \ell \nu)$ agrees with the experimental data.

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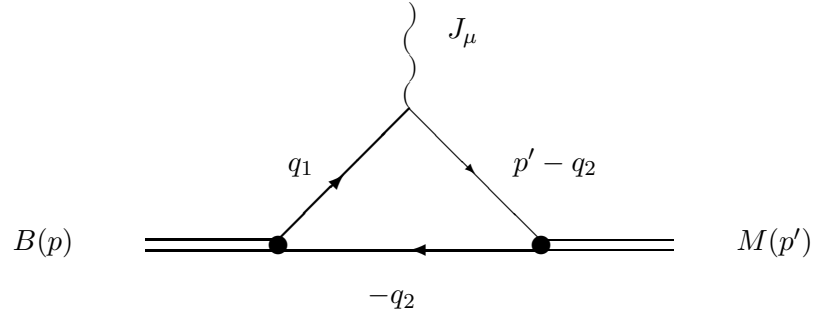


Figure 1: Quark loop diagram describing the matrix element $\langle M(p') | J^\mu | B(p) \rangle$; M is a light pseudoscalar meson, $J_\mu = \bar{q}' \Gamma^\mu b$ is the current inducing the decay and Γ^μ is a combination of Dirac matrices.

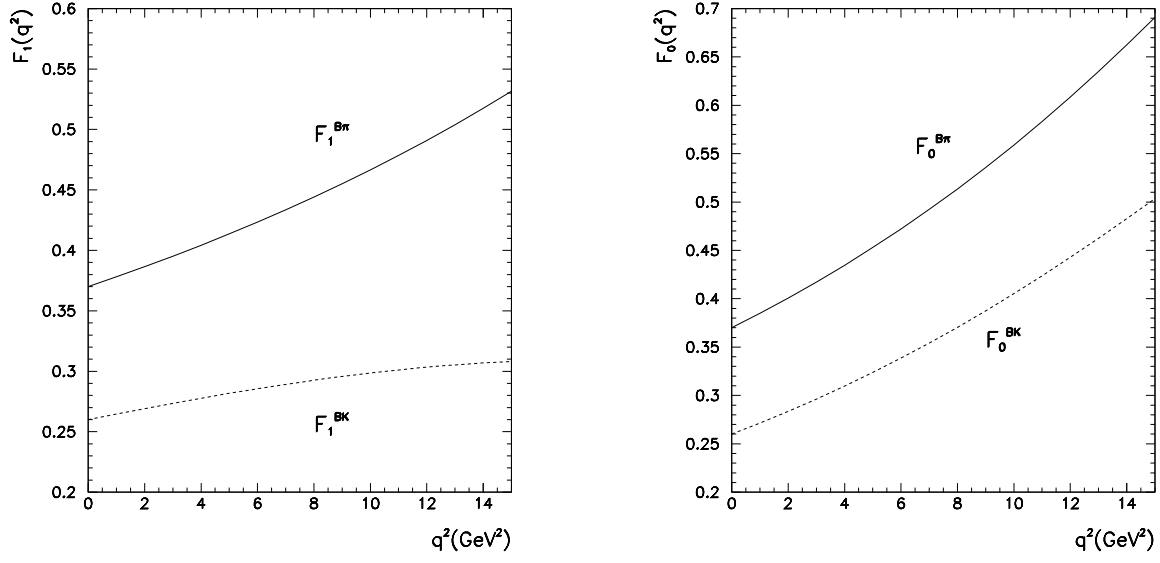


Figure 2: $F_0(q^2)$, $F_1(q^2)$ for $B \rightarrow \pi$ and $B \rightarrow K$ transitions.

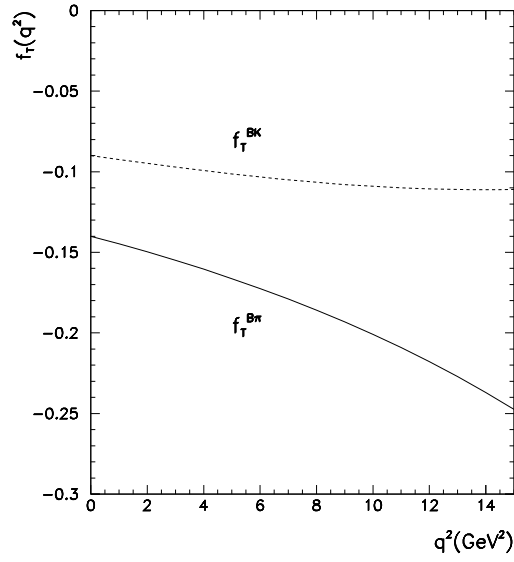


Figure 3: $f_T(q^2)$ for $B \rightarrow \pi$ and $B \rightarrow K$ transitions.